

Calculators are not allowed  
Answer the following questions

1. Let  $y = 5x^2 - 3$  and  $\Delta x$  be an increment of  $x$ .

(a) Find general formulas for  $\Delta y$  and  $dy$ .    (3 points)

(b) If  $x$  changes from 3 to 3.1, then find  $dy$ .    (3 points)

2. (a) Find an equation of the normal line to the graph of the following equation

$$x \sin y + xy + y - x = 0.$$

at the point whose  $x$ -coordinate is 0.    (5 points)

(b) Find the extrema of

$$f(x) = 2 \sin x - x \quad \text{on } [0, \pi].$$

( $\sqrt{3} \approx 1.73$  and  $\pi \approx 3.141$ ).    (5 points)

3. (a) A point  $P$  moves on the circle  $x^2 + (y-2)^2 = 5$ , in such a way that its  $y$ -coordinate increases at a rate of 4 units per second. How fast is its  $x$ -coordinate changing when  $x = 1$ ?    (5 points)

(b) State Rolle's theorem. Does this theorem apply to  $f(x) = \sqrt{9-x^2}$  on  $[-3, 3]$ ? If yes, find  $c$  which satisfies the conclusion of the theorem, and if not explain why not.    (2+3 points)

4. Let

$$f(x) = \frac{x}{\sqrt[3]{x^2-1}}$$

(a) Show that  $f'(x) = \frac{x^2-3}{3\sqrt[3]{(x^2-1)^4}}$     (3 points)

(b) Find the vertical and horizontal asymptotes for the graph of  $f$  (if any).    (3 points)

(c) Find the intervals on which  $f$  is increasing or decreasing and find the local extrema of  $f$  (if any).    (3 points)

(d) Given that  $f''(x) = \frac{2x(9-x^2)}{9\sqrt[3]{(x^2-1)^7}}$ , find the intervals on which the graph of  $f$  is concave upward or concave downward, and find the points of inflection (if any).    (3 points)

(e) Sketch the graph of  $f$ .    (3 points)

(Good Luck)